15:10:59 OCA PAD INITIATION - PROJECT HEADER INFORMATION 04/24/90

Active

Project #: E-21-F64 Cost share #: Rev #: 0
Center # : 10/24-6-R6933-0A0 Center shr #: OCA file #:
Work type : RES
Contract#: 26921A Mod #: Document : AGR
Prime #: Contract entity: GTRC

Subprojects ? : N
Main project #:

Project unit: EE Unit code: 02.010.118
Project director(s):
FAN K M EE (404)894-2901

Sponsor/division names: UNIV OF MARYLAND / COLLEGE PARK, MD
Sponsor/division codes: 400 / 032

Award period: 890401 to 900331 (performance) 900430 (reports)

Sponsor amount	New this change	Total to date
Contract value	2,376.00	2,376.00
Funded	2,376.00	2,376.00
Cost sharing amount		0.00

Does subcontracting plan apply ?: N

Title: ROTORCRAFT FLIGHT CONTROL SYSTEM DESIGN METHODOLOGIES

PROJECT ADMINISTRATION DATA

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SYSTEMS RESEARCH CENTER UNIVERSITY OF MARYLAND COLLEGE PARK, MD 20742	OFFICE OF RESEARCH ADMIN. AND ADVAN. UNIVERSITY OF MARYLAND 2100 LEE BUILDING COLLEGE PARK, MD. 20742-5141

Security class (U,C,S,TS) : U ONR resident rep. is ACO (Y/N): N
Defense priority rating : N/A N/A supplemental sheet
Equipment title vests with: Sponsor X GIT
NONE PROPOSED.
Administrative comments -
INITIATION OF PROJECT. NO FUNDS ARE AUTHORIZED FOR THE PURCHASE OF EQUIPMENT.
SEE ART. 9 OF AGREEMENT.

SR927-2

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 06/14/90

Project No. E-21-F64

Center No. 10/24-6-R6933-0A0

Project Director FAN K M

School/Lab EE

Sponsor UNIV OF MARYLAND/COLLEGE PARK, MD

Contract/Grant No. 26921A

Contract Entity GTRC

Prime Contract No.

Title ROTORCRAFT FLIGHT CONTROL SYSTEM DESIGN METHODOLOGIES

Effective Completion Date 900331 (Performance) 900430 (Reports)

Closeout Actions Required:	Y/N	Date Submitted
Final Invoice or Copy of Final Invoice	Y	
Final Report of Inventions and/or Subcontracts	N	
Government Property Inventory & Related Certificate	N	
Classified Material Certificate	N	
Release and Assignment	N	
Other	N	

Comments

Subproject Under Main Project No.

Continues Project No.

Distribution Required:

Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
Procurement/Supply Services	Y
Research Property Managment	Y
Research Security Services	N
Reports Coordinator (OCA)	Y
GTRC	Y
Project File	Y
Other	N
	N

E 21-F64

Final Report

on

Rotorcraft Flight Control System Design Methodologies

by

Ko-Hui Michael Fan

School of Electrical Engineering

Georgia Institute of Technology

Atlanta, GA 30332

Project No. : E-21-F64

Joint work with J Barlow, M. Takahashi, M.Tischler, A. Tits and N.-K. Tsing

On the design of decoupling controllers for advanced rotorcraft in the hover case [1]

J. Barlow[†], *M. Fan*[‡], *M. Takahashi*[*],
M. Tischler[*], *A. Tits*[#] *and N.-K. Tsing*[#]

ABSTRACT

1. Introduction

A methodology is proposed that can account for various types of concurrent specifications: stability, decoupling between longitudinal and lateral modes, handling quality, and physical limitations of the swashplate. This is achieved by synergistic use of analytical techniques (Q-parametrization of all stabilizing controllers, transfer function interpolation) and advanced numerical optimization techniques.

In Section 2 below, we briefly introduce a simplified model of a rotorcraft in hover. In Section 3, various design specifications are discussed. Finally, in Section 4, our design methodology is outlined.

2. A Simple Model

An overall block diagram of the closed-loop system is represented in Figure 1. The transfer function $P(s)$ models the rotor and airframe dynamics (see Figure 2 for more detail). The input variables $\delta_{s\theta}, \delta_{sc}, \delta_{s\phi}$ and $\delta_{s\psi}$ represent respectively the longitudinal, collective and lateral displacements of the swashplate, and the position of the tail rotor. The output variables are the pitch rate (q), longitudinal velocity (u), vertical velocity (w), roll rate (p), yaw rate (r), lateral velocity (v), pitch angle (θ), roll angle (ϕ), and yaw

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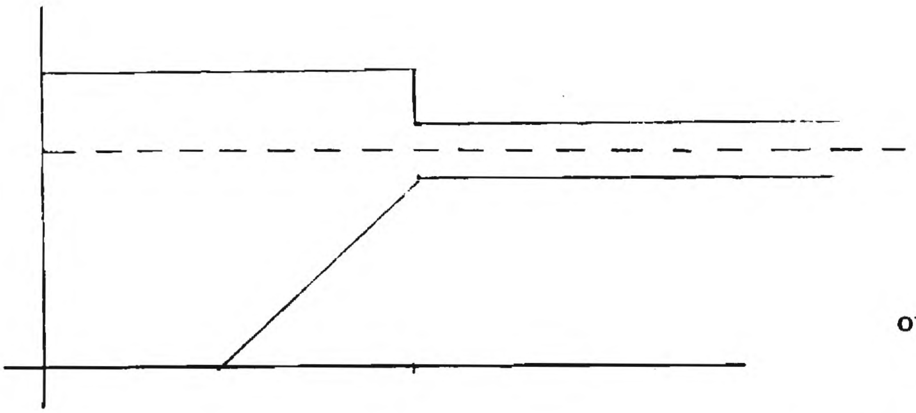
angle (ψ). The delay block $D_e(s)$ models the hydraulic transmission. The command input δ consists of pitch command (δ_θ), collective command (δ_c), roll command (δ_ϕ), and yaw command (δ_ψ). The rotorcraft is to be controlled by a two-parameter controller $C(s)$.

3. Design Specifications

A wide range of specifications, both in time- and frequency domain, are to be satisfied. First, the closed-loop system is to be internally stable. Second, to the extent possible, it is desired to decouple the various longitudinal and lateral modes and to suitably approach specified step responses. Specifically, the pitch command should mostly affect the pitch angle (and pitch rate) and the longitudinal velocity; the collective command should mostly affect vertical velocity; the roll command should mostly affect the roll angle (and roll rate) and the lateral velocity; and the yaw command should affect mostly the yaw angle (and yaw rate). The "diagonal" responses should exhibit desirable characteristics. For example, specifications for a pitch command step input of 5 inches at the pilot stick could be as follows (for the UH-60 rotorcraft).

- $\theta(t)$ should lie in the shaded area on Figure 3.



overshoot $=$ 35%

settling $=$ 5%

steady state (s.s.) $= \frac{1}{2}$ rad.

- $\max_t |\phi(t)| \leq$ 5% of s.s. of $\theta(t)$,

  $\max_t |\psi(t)| \leq$ 5% of s.s. of $\theta(t)$,

  $\max_t |w(t)| \leq$ 5% of s.s. of $u(t)$,

  $\max_t |v(t)| \leq$ 5% of s.s. of $u(t)$.

Third, displacements and rates at the swashplate must not exceed given physical limitations, e.g., for step input of 5 inches at the pitch stick (UH-60),

$$\max_t |\delta_{s\theta}(t) + \delta_{sc}(t)| \leq 10,$$

$$\max_t |\delta_{s\theta}(t) + \delta_{s\phi}(t)| \leq 10,$$

$$\max_t |\delta_{s\theta}(t)| \leq 10,$$

$$\max_t |\delta_{sc}(t)| \leq 10,$$

$$\max_t |\delta_{s\phi}(t)| \leq 10,$$

$$\max_t |\delta_{s\psi}(t)| \leq 10,$$

$$\max_t |\dot{\delta}_{s\theta}(t) + \dot{\delta}_{sc}(t)| \leq 10,$$

$$\max_t |\dot{\delta}_{s\theta}(t) + \dot{\delta}_{s\phi}(t)| \leq 10,$$

$$\max_t |\dot{\delta}_{s\theta}(t)| \leq 10,$$

$$\max_t |\dot{\delta}_{sc}(t)| \leq 10,$$

$$\max_t |\dot{\delta}_{s\phi}| \leq 10,$$

$$\max_t |\dot{\delta}_{s\psi}(t)| \leq 10.$$

Fourth and finally, it is desired to best abide by the military handling quality specifications (MIL-SPECS). Specifically, the bandwidth ω_{BW} and phase delay τ_p (see, [1., pp. 11-12 .]) of the transfer function from δ_θ to θ should yield a point corresponding to level 1 in figure 4. Similar specifications are to be achieved for collective, roll and yaw inputs.

4. Methodology

It is now well known that all linear time-invariant (dynamic) controllers $C(s)$ can be put in one-to-one correspondence with all stable transfer functions $Q(s)$ by means of Youla's Q-parametrization (see, e.g., [2., pp. 141-146 .]). In fact, if $N(s)D^{-1}(s)$ is a right coprime stable factorization of the plant $G(s) = P(s)D(s)$, and if $C(s)$ is a stabilizing controller for the plant, then the closed loop transfer function from δ to y is given by $N(s)Q(s)$, and the closed loop transfer function from δ to the swashplate displacements δ is given by $D_e(s)D(s)Q(s)$, where $Q(s)$ is the stable transfer function corresponding to $C(s)$. The stability specification is thus "eliminated" and it remains to determine $Q(s)$ so as to satisfy the other specifications. Next, the model following and decoupling specifications can be formulated as an augmented model following problem. In view of the diagonal responses and the decoupling specifications, a "desired" closed loop transfer function $T(s)$ from δ to y is formulated:

			0
	0	0	
0	g_2	0	
			0
			g_4
0	0		
g_1		0	
0		g_3	
0		0	

Here, a "0" entry of $T(s)$ means the corresponding scalar transfer function is closed to 0; the g_i's are transfer functions for the "diagonal" responses; the void entries of T are not of concern at this stage of design and thus are not specified. Our objective is then to find a stable transfer function $Q(s)$ such that $N(s)Q(s)$ is close to $T(s)$ and the MIL-SPECS are met as well. It turns out that the four columns of $Q(s)$, each corresponds to a different control channel, can be designed independently. For example, to obtain a solution for the first column of $Q(s)$, which corresponds to the pitch channel and is denoted by $Q_1(s)$, we first delete the unspecified entries of the first column of $T(s)$ to form a 5×1 transfer function $\tilde{T}_1(s) = \begin{bmatrix} 0 \\ 0 \\ g_1 \\ 0 \\ 0 \end{bmatrix}$. Also, delete the corresponding rows of $N(s)$ to form a 5×4

transfer function $\tilde{N}_1(s)$, so that the rows of $\tilde{N}_1(s)$ are the 3, 6, 7, 8, 9th rows of $N(s)$. The desired transfer function $g_1(s)$ is modeled by a second order function

$$g_1(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} .$$

Choose a fixed grid of sampling frequencies $\omega_1, \ldots, \omega_\ell$, and let $H = \operatorname{diag}(h_1, h_2, h_3, h_4, 1)$ be a weighting matrix. For each $i = 1, \ldots, \ell$, obtain the least square fit solution $X_i \in \mathbb{C}^{4\times 1}$ of the linear equation

$$H\tilde{N}_i(j\omega_i)X_i = H\tilde{T}_1(j\omega_i) .$$

We require $Q_1(j\omega_i)$ to be close to X_i for $i = 1, \ldots, \ell$. Let $Q_1(s)$ be of order $2n$ and of the form $Q_1(s) = C_1(sI - A_1)^{-1}B_1 + D_1$, where

$$A_1 = \begin{bmatrix} 0 & 1 & & & & & \\ p_1 & p_2 & & & & & \\ & & 0 & 1 & & & \\ & & p_3 & p_4 & & & \\ & & & & \ddots & & \\ & & & & & 0 & 1 \\ & & & & & p_{2n-1} & p_{2n} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ b_1 \\ 1 \\ b_2 \\ \vdots \\ 1 \\ b_n \end{bmatrix} .$$

Here $2n$ is fixed, and $p_1, \ldots, p_{2n} < 0$ in order that $Q_1(s)$ is stable. Suppose $E_i = (j\omega_i I - A_1)^{-1}B_1$ for $i = 1, \ldots, \ell$. Let $K = \operatorname{diag}(k_1, k_2, \ldots, k_{\ell-1}, 1)$ be a weighting matrix. Then $C_1(\in \mathbb{R}^{4\times 2n})$ and $D_1(\in \mathbb{R}^{4\times 1})$ are obtained as the least square fit solution of the linear equation

$$(C_1 D_1)\begin{pmatrix} E_1 & E_2 & \cdots & E_\ell \\ 1 & 1 & \cdots & 1 \end{pmatrix} K = [X_1 X_2 \ldots X_\ell]K .$$

For the problem under consideration, by varying the design parameters $\omega_n, \xi, \omega_1, \ldots, \omega_\ell$, $h_1, \ldots, h_4$, $p_1, \ldots, p_{2n}$, $b_1, \ldots, b_n$, $k_1, \ldots, k_{\ell-1}$, a family of solution for $Q_1(s)$ can then be obtained. Similar procedures are followed to obtained the other columns of $Q(s)$.

Finally, our central tool is numerical optimization. Semi-infinite optimization techniques allow to tackle specifications to be satisfied at every instant in time, such as those corresponding to physical limitations. CONSOLE [3], an interactive optimization-based design software package recently developed at the University of Maryland makes such techniques readily available, while allowing the designer to explore tradeoffs between the various specifications.

References

[1] M.B. Tischler, "Digital Control of Highly Augmented Combat Rotorcraft ," NASA TM 88346, 1987.

[2] M. Vidyasagar, *Control System Synthesis - A Factorization Approach*, MIT Press, 1985.

[3] M.K.H. Fan, L.-S. Wang, J. Koninckx & A.L. Tits, "Software Package for Optimization-Based Design with User-Supplied Simulators," *IEEE Control System Magazine* 9 (1989).